Anomaly in symmetry broken phase and crystalline SPTs

Anomaly matching, crystalline equivalence principle, and LSM map

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Outline

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2. SSB = crystalline
3. Applications

Symmetry breaking phase set up

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- In the symmetry breaking phase, ϕ condenses and we pick up a vev $\langle\phi\rangle \neq 0$. We will consider the theory Z with a varying background $\langle\phi(x)\rangle$.
- Equivalently, we can view $Z[\phi(x)]$ as a family of field theories with parameter $\phi \in V_\rho$. And we are allowed to change the parameter ϕ across space.

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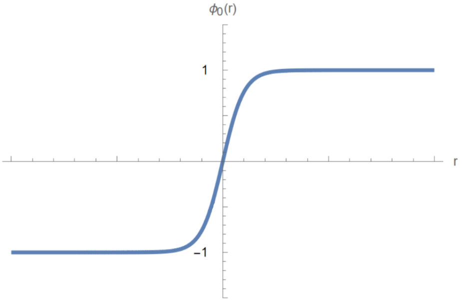

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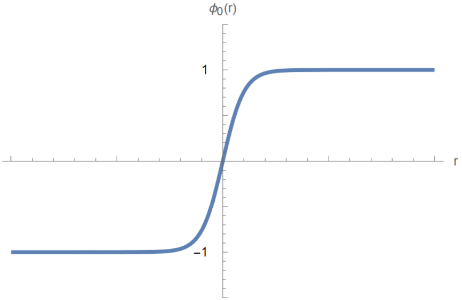

- Note this creates a topological defect at the origin $x^{D-2} = x^{D-1} = 0$.

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- However, note G acts on ϕ the same way as a space rotation in the (x^{D-2}, x^{D-1}) coordinates.
- Let R_α be the spatial rotation in (x^{D-2}, x^{D-1}) coordinates by angle α , then we see that

$$U_\alpha R_{-\alpha}\phi = \phi$$

is a symmetry of the system $Z[\phi]$. Let us denote this twisted symmetry group as G', it is a crystalline symmetry as it acts on space.

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- Let's see some applications of this point of view.

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- In upcoming work, viewing the crystalline phase as the symmetry breaking phase, we give an intuitive explanation of CEP as going between G and G'.

$$\text{D dim, G internal} \xleftarrow[CEP]{\simeq} \text{D dim, G' crystalline}$$

Application II: Anomaly matching in SSB, and LSM

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- There is an anomaly matching map, called the Smith homomorphism, that maps the defect anomaly to the bulk anomaly. The anomaly matching condition states the defect anomaly maps to the bulk anomaly under Smith homomorphism.

D dim, G internal

anomaly matching ↑

D-k dim, G' internal

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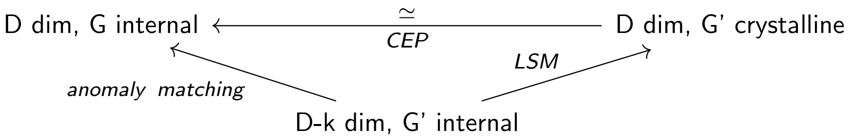

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